Fourth Semester B.E. Degree Examination, June/July 2019
Signals and Systems

Time: 3 hrs.   Max. Marks:100

Note: *Answer any FIVE full questions, selecting*
at least TWO full questions from each part.

PART – A

1 a. Define signal and system with examples. **(06 Marks)**

   b. Prove that

   i) $\int_{-a}^{a} x(t)dt = \alpha\int_{o}^{a} x(t)dt$ If x(t) is even   ii) $\int_{-a}^{a} x(t)dt = 0$ If x(t) is odd.   **(06 Marks)**

   c. For the following system, determine whether the system is   a) Linear   b) Time invariant
      c) Memory less   d) Causal.
      i) $T[x(n)] = g(n)\,x(n)$ ii) $T[x(t)] = e^{x(t)}$ **(08 Marks)**

2 a. The trapezoidal signal as shown in Fig.Q.2(a) applied to differentiator defined by
      $y(t) = \dfrac{d}{dt}x(t)$

   i)   Find the resulting output of differentiator   ii) Find the total energy of y(t).   **(06 Marks)**

Fig.Q.2(a)
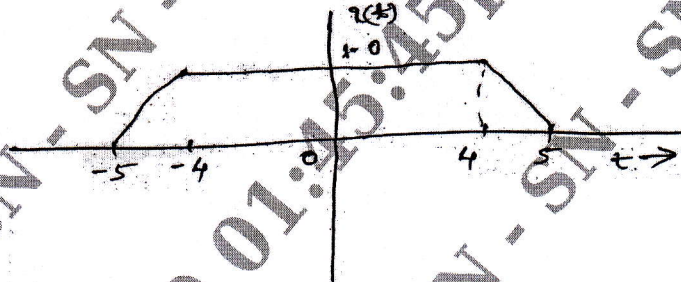
   b. Find the discrete-time convolution sum of $y(n) = \beta^n u(n) * \alpha^n u(n)$ $|\alpha| < 1 : |\beta| < 1$.   **(06 Marks)**

   c. Consider a continuous-time LTI system with unit impulse response. h(t) = u(t) and input
      $x(t) = e^{-at} u(t)$ $|a| > o$. Find the output y(t).   **(08 Marks)**

3 a. Prove that
      i) $x(n) * h(n) = h(n) * x(n)$
      ii) $x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$ **(08 Marks)**

   b. Find the output of the system given by the differential equation:
      $\dfrac{d^2 y(t)}{dt^2} + 5\dfrac{dy(t)}{dt} + 4y(t) = \dfrac{dx(t)}{dt}$ with $y(0) = 0$ $\dfrac{dy(t)}{dt}\Big/_{t=0} = 1$ and $x(t) = e^{-2t} u(t)$.

   (06 Marks)

   c. Draw the direct form I and direct form II implementation of the following system shown
      below:s

      i) $\dfrac{d^3 y(t)}{dt^3} + 2\dfrac{dy(t)}{dt} + 3y(t) = x(t) + 3\dfrac{dx(t)}{dt}$

      ii) $y(n) - \dfrac{1}{4}y(n-1) + \dfrac{1}{8}y(n-2) = x(n) + \dfrac{1}{2}x(n-2)$ **(06 Marks)**

4 a. Determine the DTFS of the signal

    i) $x(n) = \cos\left(\frac{\pi}{3} n\right)$

    ii) $x(n) = \sum_{M=-\infty}^{\infty} \delta(n - 4m)$ **(08 Marks)**

  b. Determine the FS representation for the signal

    i) $x(t) = \cos 4t + \sin 8t$ ii) $x(t) = e^{-t}$

                                          $-1 < t < 1$

                                          $T = 2$ **(08 Marks)**

  c. Prove the following properties:

    i)   If $x(t) \xleftrightarrow{FS, w_o} x(k)$ then $y(t) = x(t - t_o) \xleftrightarrow{FS, w_o} y(k) = e^{-jkw_o t_o} x(k)$

    ii)   If $x(t) \xleftrightarrow{FS, W_o} x(k)$ then $y(t) = e^{jk_o w_o t} x(t) \xleftrightarrow{FS, w_o} y(k) = x(k - k_o)$.   **(04 Marks)**

PART – B

5 a. Compute DTFT of the following signals:

    i) $x(n) = 2^n u(-n)$ ii) $x(n) = a^{|n|}$ $|a| < 1$ **(08 Marks)**

  b. Find the Fourier transform of $x(t) = e^{-a|t|}$ $a \geq 0$. Draw its spectrum.   **(06 Marks)**

  c. Find the inverse Fourier transform:

    i) $x(jw) = \dfrac{5jw + 12}{(jw)^2 + 5jw + 6}$

    ii) $x(jw) = \dfrac{jw}{(2 + jw)^2}$ **(06 Marks)**

6 a. Find the relationship between: i) FT and FS   ii) DTFT and DTFS.   **(08 Marks)**

  b. Specify the Nyquist rate for each signals :

    i) $x_1(t) = \text{sinc}(200t)$ ii) $x_2(t) = \text{sinc}^2(200t)$ **(06 Marks)**

  c. Find the frequency response and impulse response of the following system:

$$\frac{d^2 y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = -\frac{dx(t)}{dt}$$ **(06 Marks)**

7 a. Determine the Z-transform, ROC, pole zero location of the following system:

    i) $x(n) = \alpha^n u(n)$

    ii) $x(n) = -\alpha^n u(-n-1)$

    iii) $x(n) = a^n \cos(\Omega_o n) u(n)$ for $\Omega_o = 2\pi$.   **(09 Marks)**

  b. Explain the properties of ROC.   **(06 Marks)**

  c. Prove that

    i) $x(n - n_o) \xleftrightarrow{z} z^{-no} x(z)$

    ii) $a^n x(n) \xleftrightarrow{z} x\left(\dfrac{z}{a}\right)$ **(05 Marks)**

8 a. Determine whether the system described below is causal and stable

$$H(z) = \frac{2z + 1}{z^2 + z - 5/16}.$$ **(06 Marks)**

  b. Consider a system described by the difference equation.

$$y(n) - 2y(n - 1) + 2y(n - 2) = x(n) + \frac{1}{2}x(n - 1)$$

    Find: i) $H(z)$ ii) $h(n)$ iii) Stability.   **(08 Marks)**

  c. What is unilateral Z-transform and prove its time shifting property.   **(06 Marks)**

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